

ECE 124 - W22  
Digital Circuits and Systems  
Full Course Notes

With Prof Otman Basir

---

My notes cover all the course material; at least, everything on the final was on here. Hope they help!

---

[Josiah Plett](#)

# ECE 124 Notes 1

★  $(x+y)(x+z) = x+yz$  ★  
★  $X+Xy = X$  ★

computer-aided design

① Intro • We'll use CAD & FPGA's (Field Programmable Logic Devices) • Everything's Binary

0 - min-term  
1 - max-term

Combinational Circuits • Don't care about time  
Sequential Circuits • Have a sense of time

② Analog/Digital, Design, & Number types • Motherboard: ties a computer's components together.

Analog: the input/output IS the number (Physical)

Digital: stuff is in BINARY (logical).

Binary (2), Octal (8), Hex (16)  
Bi → Oct: groups of 3  
Bi → Hex: groups of 4

Complement:

How much you need to add to get to the maximum single digit...

Within Radix R: FOR NEGATIVE NUMBERS  
R's complement:  $R^n - N$  for  $n \neq 0$  ( $0 \rightarrow 0$ )

Another way: Each digit is  $(r-1) - \langle \text{digit} \rangle$ , then add 1.

$(r-1)$ 's complement: Just don't add 1

## Design Cycle

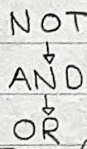
- Learn what the product requires.
- Specifications & initial design
- Is the design correct? NO
- Prototype implementation and testing
- Meets specifications? NO  
*is done! congrats!*

Name	Operators	Gates
AND	$f \cdot f$	
OR	$f + f$	
NOT	$\bar{f}, f', !f, \sim f$	
NAND		
NOR		

③ Switches

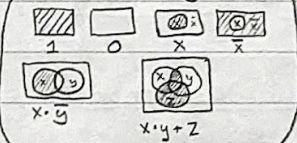
$L(x) = X$  is a Logic Function

Precedence:



Groups of gates: "Logic Circuit"

Venn Diagrams



## Boolean Algebra

Axioms: AND  $0 \cdot 0 = 0$ , OR  $0 + 0 = 0$ , NOT  $x = 0 \Rightarrow \bar{x} = 1$

1-Variable

Theorems:  $x = x \cdot 1 = x \cdot x + 0 = x + x = \bar{\bar{x}}$   
 $0 = x \cdot \bar{x} = x \cdot 0$ ,  $1 = x + 1 = x + \bar{x}$

2- or 3-Variable

Theorems:  $x \cdot y = y \cdot x$ ,  $x + y = y + x$  } Commutativity  
 $x(y+z) = (x \cdot y) + z$  } Associativity  
 $x + (y+z) = (x+y) + z$   
 $x \cdot (y+z) = x \cdot y + x \cdot z$  } Distributivity  
 $x + x \cdot y = x$  } Absorption  
 $x(x+y) = x$   
 $x \cdot y + x \cdot \bar{y} = x$  } Combination  
 $(x+y) \cdot (x+\bar{y}) = x$   
 $\bar{x} \cdot y = \bar{x} + \bar{y}$  } DeMorgan's  
 $x + \bar{x} \cdot y = x + y$  } Theorem  
 $x \cdot (\bar{x} + y) = x \cdot y$   
 $x \cdot y + y \cdot z + \bar{x} \cdot z = x \cdot y + \bar{x} \cdot z$  } Consensus  
 $(x+y) \cdot (y+z) \cdot (\bar{x}+z) = (x+y) \cdot (\bar{x}+z)$

④ Synthesis

• Minterm: each variable appears once:  $5 | 1 0 1 \rightarrow m_5 = x_1 \bar{x}_2 x_3$   
• Maxterm: each variable appears once:  $6 | 1 1 0 \rightarrow M_6 = \bar{x}_1 + \bar{x}_2 + x_3$

Sum-of-Products (form)

"AND" each minterm with its function value, and "OR" those together:

x	y	f(x,y)	minterm
0	0	1	$m_0 = \bar{x}\bar{y}$
0	1	1	$m_1 = \bar{x}y$
1	0	0	$m_2 = x\bar{y}$
1	1	1	$m_3 = xy$

$f(x,y) = m_0 \cdot 1 + m_1 \cdot 1 + m_2 \cdot 0 + m_3 \cdot 1$   
 $= \bar{x}\bar{y} + \bar{x}y + xy$

Product-of-Sums (form)

"AND" each maxterm with NOT its function value, and "AND" those together:

The same function as left becomes:  
 $f(x,y) = M_0 \cdot \bar{1} \cdot M_1 \cdot \bar{1} \cdot M_2 \cdot 0 \cdot M_3 \cdot \bar{1}$   
 $= M_2 = (\bar{x} + y)$

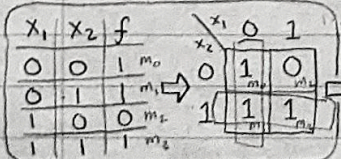
## ⑤ Karnaugh Maps

↳ ≤ 5 inputs

Canonical & Standard Forms:

- Sum of Products
- Product of sums

KMaps: alternative to truth table



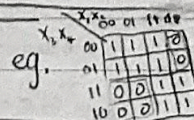
Minimum SOP:

$f = \bar{x}_1 + x_2$

Minimizing SOP Representations

- ① Select rectangles (product terms) with as many 1's as you can ( $2^n$ ,  $n \in \mathbb{Z}$ )
- ② Cover all the 1's; it's solid to cover a 1 multiple times

Minimizing POS: • surround 0's  
• AND the sumterms



$f = (\bar{x}_1 + x_2 + x_3)(\bar{x}_1 + \bar{x}_3)$

# ECE 124 Notes 2

$X_n \rightarrow$  uncomplemented (positive) literal  
 $X: 0(n) \rightarrow$  don't-cares

## ⑥ Function Minimization

$f = \bar{x}w + yw + \bar{z}yx \Rightarrow$  COST: 1OR + 3AND + 10GATEINPUTS = 14 COST

★ We can always form a cover with only prime implicants ★

### XOR & XNOR

$f_{XOR} = X_1 \oplus X_2$   
 $= \bar{X}_1 X_2 + X_1 \bar{X}_2$

$f_{XNOR} = X_1 \odot X_2$   
 $= \bar{X}_1 \bar{X}_2 + X_1 X_2$

TRY TO USE DURING MINIMIZATION!

## ⑦ Implementations

SOP: -plane of NOT gates  
 -plane of AND gates  
 -a single big OR gate

POS: -NOT gates  
 -OR gates  
 -an AND gate



• Product term: any AND of literals

### STANDARD SOP & POS

no requirement for terms to be min or max; instead, only any P or S terms!

## Terminology

- Completely Specified: every input has a defined (specific) output.
- Incompletely Specified: opposite
- Sets of minterms:
  - on-set: output is 1
  - off-set: output is 0
  - dc-set: output is X (don't care)

## More Terminology

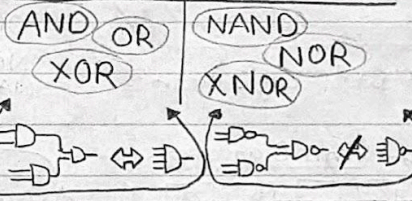
- Implicant: product term with outputs always 1 (rectangle in k-map)
- Prime Implicant: removing a literal would result in a non-implicant
- Cover: collection of implicants that cover every value of 1
- Essential Prime Implicant: if its the only prime implicant that can include any specific minterm

NAND	NOR	XOR	XNOR
$\overline{xyz}$	$\overline{x+y+z}$	$x \oplus y$	$x \odot y$

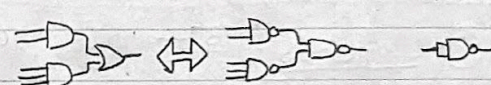
$X_1 \oplus X_2 \oplus X_3 \oplus X_4 \rightarrow$  "odd operation"  
 $(X_1 + X_2 + X_3 + X_4) \% 2$

BUFFER (boosting signal strength)

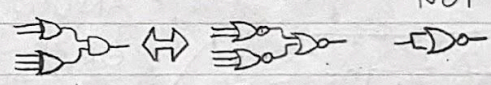
### Associative | NOT Associative



### SOP with NAND



### POS with NOR



## ⑧ CAD & Physics

- Computer Aided Design (& Drafting)
- VHDL
- VHSIC Hardware Description Language
- VHSIC
- Very High Speed Integrated Circuit

- ★ Variable names are CASE INsensitive
- ★ Concurrency: everything in parallel

### VHDL Circuit Description

- Entity Declaration: inputs & outputs
  - ↳ describes "outside world" interface
- Architecture Definition
  - ↳ describes the circuit's implementation

### DATA OBJECTS

- signals
- constants
- variables

See Slides For Syntax

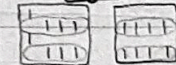
IN OUT INOUT BUFFER

### TRANSISTORS AS SWITCHES

x="low" x="high"

(Type of transistor)  
 MOSFET  
 Metal Oxide Semiconductor Field-Effect Transistor  
 (n-channel or p-channel)

### Functional Decomposition (eg 2)



$g = x_1 + x_2 + x_5$

## ⑨ Tabular Method

### Functional Decomposition

If  $f(x_1, \dots, x_n) = \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3 + \bar{x}_1 \bar{x}_2 \bar{x}_3$

Then  $f = (\bar{x}_1 x_2 + x_1 \bar{x}_2) x_3 + (x_1 x_2 + \bar{x}_1 \bar{x}_2) x_4$

Let  $g(x_1, x_2) = \bar{x}_1 x_2 + x_1 \bar{x}_2$  so  $g'(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2$

$f = g x_3 + g' x_4$

### Quine-McCluskey Method

- eliminate literals with absorption
- Minimum # of prime implicants (chart)

CAD

TABULAR METHOD

EXAMPLE

### Minterm List (list 1)

- List all applicable minterms
- Group them based on # of 1's in binary representation

$f = \sum m(0, 4, 8, 10, 11, 12, 13, 15)$

# of 1's	$m_i$	4-literal Implicants
0	0	0000
1	4	0100
1	8	1000
2	10	1010
2	12	1100
3	11	1011
3	13	1101
4	15	1111

### Prime Implications (list 2)

- Combine implicants who differ by 1 with preceding.
- Check off terms of list 1 that appear in list 2

combination	3L-Implicants
0, 4	0X00
0, 8	X000
8, 10	10X0
4, 12	X100
8, 12	1X00
10, 11	101X
12, 13	110X
11, 15	1X11
13, 15	11X1

### List 3

- Same like list 2, but make sure x's align.

we would repeat N times if we could

### Essential PIs

- PI: not checked off
- EPI: only  in a column
- FINAL ANSWER: Sum the EPIs! (min cost!)

PIs	0	4	8	10	11	12	13	15
$P_1 = 10X0$				✓	✓			
$P_2 = 101X$						✓	✓	
$P_3 = 110X$							✓	✓
$P_4 = 1X11$								✓
$P_5 = 11X1$								✓
$P_6 = XX00$	✓	✓	✓					

# ECE 124 Notes 3

$\oplus \leftrightarrow$  XOR subtraction: addition of two's complement form

## 10 Petrick's Method (branch & bound method)

- Form a function that's true when all columns are covered
  - Make a SUM of PI's that cover the minterms
  - AND all those sums together
- Reduce that function algebraically, to a min SOP.
  - $(x+y)(x+z) = x+yz$  &  $x+xy = x$
  - Be aware: each product term represents a solution!
- Determine minimum solutions (fewest # of PI's & literals)
- Choose the term(s) with min # of literals; write out the corresponding minimum sum of PI's!

ROM  $\rightarrow$  Read-only Memory (unchangeable)  $m$  inputs  $n$  outputs }  $2^m \times n$  ROM  
 PROM  $\rightarrow$  Programmable Memory

## 12 Combinatorial Circuits

### Binary half-adder

x	y	sum	cout
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

sum =  $\bar{x}y + y\bar{x}$   
 cout =  $xy$

### Binary full-adder

sum =  $x \oplus y \oplus C_{in}$   
 cout =  $xy + C_{in}(x \oplus y)$

### Carry look-ahead adder

- p(i): propagate; sum
- g(i): generate; cout
- use functional substitution

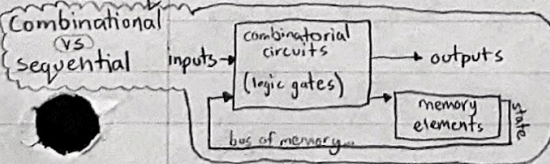
$C_1 = g_0 + C_0 p_0$   
 $C_2 = g_1 + C_1 p_1 = g_1 + p_1 g_0 + p_1 p_0 C_0$   
 $C_3 = g_2 + C_2 p_2 = g_2 + p_2 g_1 + p_2 p_1 g_0 + p_2 p_1 p_0 C_0$   
 $C_4 = \dots$

Performance: 3 units

### Magnitude Comparators

- equality of bits:  $e_i = a_i b_i + \bar{a}_i \bar{b}_i$
- overall equality:  $(A=B) = e_n e_{n-1} \dots e_0$
- inequality of bits: conceptual algorithm
- also, remember:  $(A < B) = (A=B) + (A > B)$

## 13 Sequential Circuits (& recognizers)



**Synchronous vs Asynchronous**  
 Synchronous: circuit behaviour is determined from knowledge of values at discrete instances in time.  
 Asynchronous: circuit behaviour is determined by signals at any instance in time, and the order the inputs change.

**CLOCK SIGNAL**  
 used to control behaviour of a circuit at discrete points in time.

### Latches - type of storage element (helps: asynchronous)

- Level sensitive; operate at 0 or 1, not  $\rightarrow$  or  $\leftarrow$  "rising edge"
- NOR SR Latch:  $(1,0)$  means  $Q=1$ ;  $(0,1)$  is storage;  $(0,0)$  means  $Q=0$ ;  $(1,1)$  is undesirable!
- SR Latch:  $(1,0)$  means  $Q=1$ ;  $(0,1)$  is storage;  $(0,0)$  means  $Q=0$ ;  $(1,1)$  is undesirable!

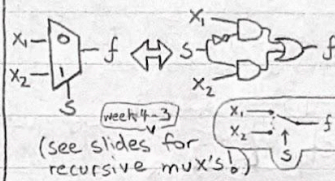
**NAND SR Latch:**  
 same as NOR but:  
 $(1,0) \rightarrow Q=0$   
 $(0,1) \rightarrow Q=1$   
 $(1,1) \rightarrow$  storage  
 $(0,0) \rightarrow$  bad!

**Gated Latch**

C	S	R	next Q value
0	x	x	no change
1	0	0	no change
1	0	1	0
1	1	0	1
1	1	1	undefined

## 11 Multi-Level Combinational Circuits (Modules)

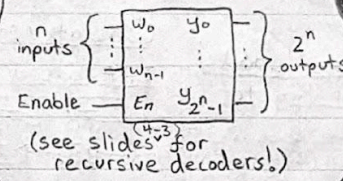
### Multiplexer (MUX)



### Function Synthesis (w/MUX)

Shannon's Expansion  
 $f(w_1, w_2, \dots, w_n) = w_1 f(0, w_2, \dots, w_n) + w_1' f(1, w_2, \dots, w_n)$   
 $f = w_1' \cdot f_{w_1'} + w_1 \cdot f_{w_1}$   
 "cofactors" of f, respect to  $w_1$   
 eg  $f_{w_1} = w_2' f_{w_1 w_2'} + w_2 f_{w_1 w_2}$  (4 INPUTS)  
 eg  $f_{w_1 w_2} = f(0, 1, w_3, \dots)$

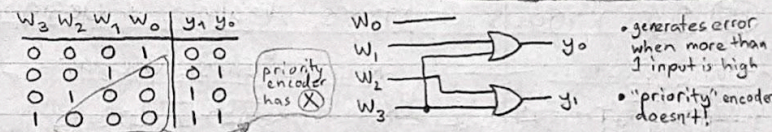
### Decoder (binary decoder)



### De-Multiplexer

$S_0 S_1$	$S_0 S_1$	$y_0$	$y_1$	$y_2$	$y_3$
00	00	x	0	0	0
01	01	0	x	0	0
10	10	0	0	x	0
11	11	0	0	0	x

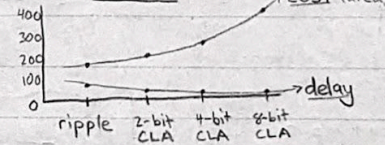
### Encoder (binary decoder)



## Performance

1 logic gate  $\equiv$  1 "unit" of delay  
 Combinatorial circuit's performance is determined by the longest path.

Tradeoff: Area vs. delay  
 8-bit adder

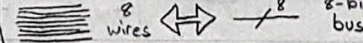


## Overflow Detection $\rightarrow$ if you need n+1 bits

Unsigned numbers: if there is a carry-out from the MSB (Most Significant Bit), then there's overflow.

## Busses

A bundle of wires that carries data someplace! "16-bit bus"

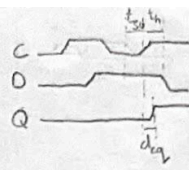


in this course we'll never have  $\geq 2$  sources driving at the same time, we use multiplexers as control!

# ECE 124 Notes 4

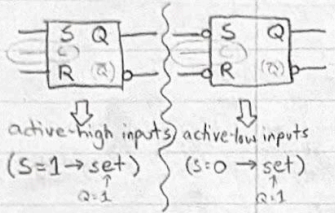
**Definition: Propagation Delays:**  
 time held stable before edge  $\rightarrow$  Setup time:  $t_{su}$   
 time held stable after edge  $\rightarrow$  Hold time:  $t_h$   
 time till output is stable  $\rightarrow$  Clock-to-output time:  $t_{cq}$

**Definition: Synchronous: same clock**  
**Asynchronous: not same with clock**

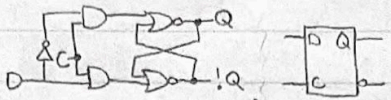


## 13.2 Latches (ct'd)

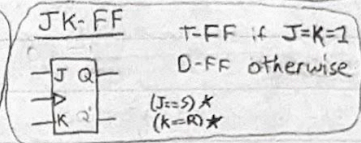
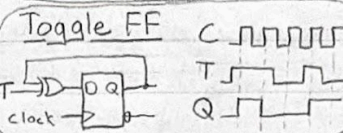
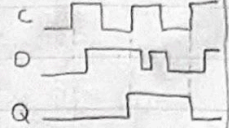
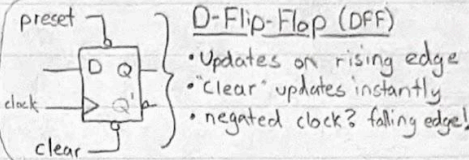
### Schematic Symbols



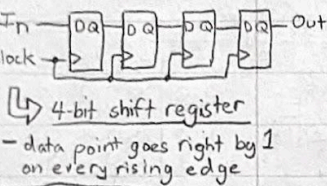
**D-latch** captures the logic level that's present on the Data line when the Control is high;  
 $C=1 \rightarrow Q=D$   
 $C=0 \rightarrow Q=$  it's previous value



## 14 Flip Flops



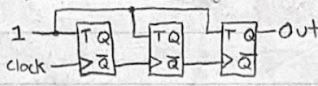
## Multi-Cell Storage Components



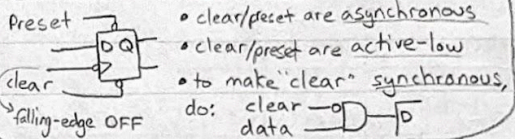
**Parallel Load Shift Register**  
 - series & parallel input (& output)

## Counters

### TFF-based sync? Counter

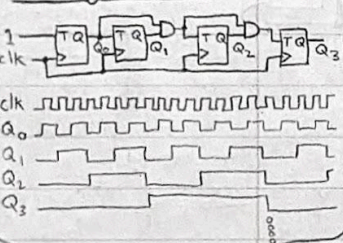


### DFF with clear/preset

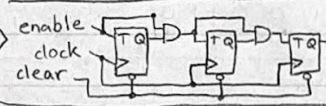


## 15 Synchronous Counters, Analysis, & Design

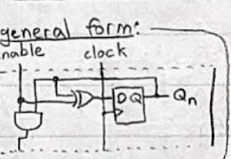
### 4-bit Sync Counter



### Now with Enable & Clear



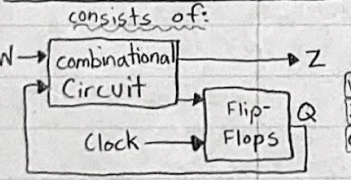
### DFF-based counters



**Parallel-load capability counter**  
 See lecture notes week-6-3 pg 6

**Modulo-N counter**  
 See lecture notes week-6-3 pg 7

### FSM-Finite State Machine

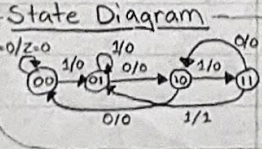


**Moore Machine**  
 FSM except Z depends only on Q (sync w/clock)  
 • Z changes synchronously

**Mealy Machine**  
 FSM except Z depends on Q and W (sync w/input)  
 • Z changes asynchronously

## 17 State/Transition Tables

Alternate form to state diagrams:



**State Table**

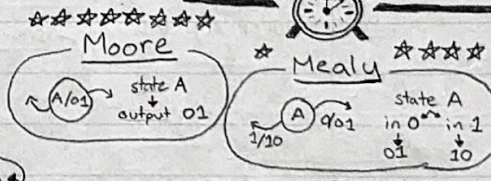
curr state $Q_0 Q_1$	next state $Q_0 Q_1$	output Z
$X=0$	$X=1$	$X=0$ $X=1$
00	01	0 0
01	10	0 0
10	01	0 1
11	00	1 1

## 16 Sequential Circuits

Types of Finite State Machines

**State Diagram**  
 • Possible states: state bubbles  
 • Possible transitions: directed edges  
 • Circuit outputs: labelled on those

**Edge-in State Diagram**  
 Indicates the transitions based on state when the rising edge returns

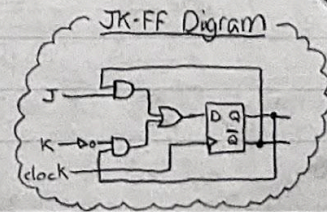


Any sequential circuit can be described with a state diagram that's either Mealy or Moore.

**Purpose of Reset:**  
 We want to know what the system's at when it starts/restarts

**Master vs Slave:** slave gives output rising edge falling edge

**XOR:** odd number of highs!



# ECE 124 Notes 5

## DESIGN

### 18 Synchronous Circuit Design

IF DFF, these are equal!



### 19 (17!) Circuit Analysis

#### Procedure:

- Identify the FFs used to hold the current state
- Identify the outputs of the circuit
- Write the logic equations for circuit outputs & FF inputs ("next state equations")
- Use the "next state equations" to derive a state table w/ next state and outputs
- (optional) Make a state diagram from the state table.

## ANALYSIS

THAT BUT BACKWARDS

Using sentences to describe to left

### 19 State Reduction

The state assignment problem...

- make the output the curr state; perhaps adding "extra bits"
- more than minimum FFs
- potentially lots of unused states

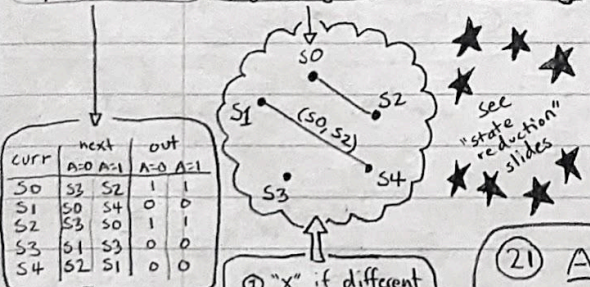
### Reduction

States are equivalent if:

- they give identical outputs
- they give same/equivalent next states

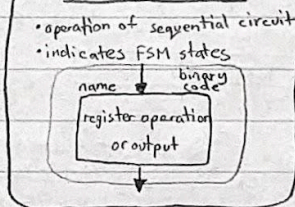
methods for reduction

Implication charts, Merger Diagrams, Partitioning



- "x" if different output values
  - "y" if definitely equivalent
  - "(so, si)" if equiv. under the case that those 2 are equivalent
- under which conditions are any of these equivalent? check!

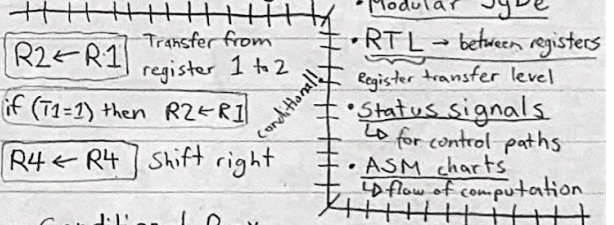
### State Box



### 20 Algorithm State Machines ASM

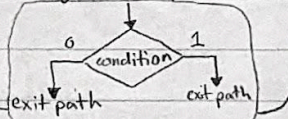
- Take advantage of registers & counters.
- use decoders & muxes for logic.

- Data Paths (X)
  - information reception + delivery
- Control Paths (D, T, JK...)
  - logic between things



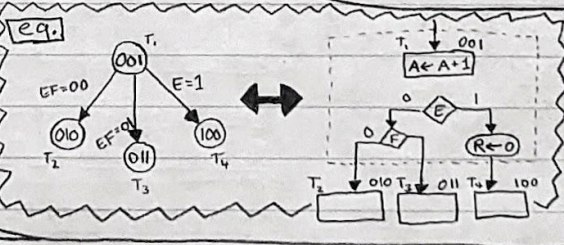
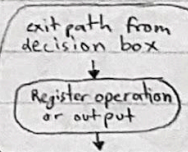
### Decision Box

- impact of input on the control system
- implemented using a magnitude comparator (MC)

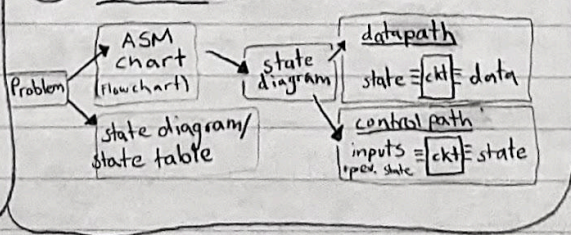


### Conditional Box

- indicates assignments following a decision box; data transfer usually



### 21 ASM 2



"Less is Moore"

↳ depends on less; i.e. not on prev. state or input

"NOR is Normal"

NOR latches are the normal ones

# ECE 124 Notes 6

transition table ← flow table

$y_1 y_2$	$Y_1 Y_2$	$Z$	$y_1 y_2$	$Y_1 Y_2$	$Z$
00	01, 0	0	a	b, 0	0
01	10, 0	0	b	c, 0	0
10	10, 0	0	c	c, 1	0
11	01, 0	0	d	b, 0	0

would be multiple based on X

↳ "Loop" through logic gates

## 22 Asynchronous Sequential Circuits

- feedback loops that resemble latches

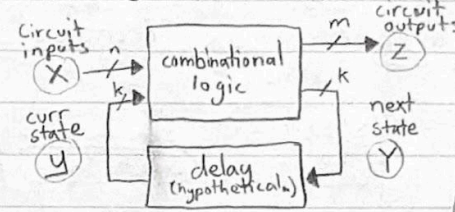


### Asynchronous Analysis

Identify: has latches? Async! or equivalent...

Generally: stable state is unpredictable

## 23 Async Circuits



• Stability: stable means steady state (unchanging).

• Fundamental Mode Operation:  
↳ only one input can change at a time  
↳ it changes only if circuit is stable

• Circle stable states in your transition tables:

curr	next	output
$y_i$	$x=0$ $x=1$ $Y_i$ $Y_i$	whatever lol
0	0 1	0
1	1 0	0

• Primitive flow table  
↳ one stable state per row

## 24 Races, Cycles, state Assignment

eg

$y_1 y_2$	$x$
00	01
00	11
01	01
01	11
10	01
10	10

00 → 11 RACE  
 $y_1$  changes first  
00 → 10 (stable)  
 $y_2$  changes first  
00 → 01 → 11 (stable)  
change simultaneously  
00 → 11 (stable)

This is a critical race.  
(different stable states)

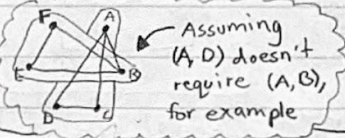
## 25 State Reduction

A and B are compatible if, for each input combo:  
1 They have same outputs AND where  
2 They have compatible next states specified ← [X]

Implication Chart

Merger Diagram

↳ choose largest "cliques" (loops)



## 26 State Assignment

Shared Row SA    MultiRow SA    One-hot SA

## 27 RAM & ROM

EEPROMS store data and Functions and Design Building blocks  
PAL: Programmable Array Logic  
↳ Products followed by sums

Erasable Electrically Programmable Read-Only Memory

"programming" means blowing fuses to give some bits = 0 - only do it once!  
- Permanent ROM  
- (K) addresses  
- (n) bits  
-  $2^k \cdot n$  info

RAM (soft) Random access memory  
READ and WRITE

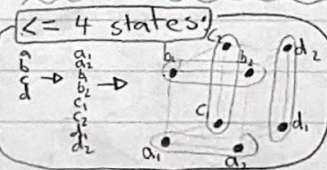
PLA Programmable Logic array implements SOP input buffers & inverters → AND → OR

PAL Programmable array logic PLA but the OR plane is fixed

CPLD → very big SPLD's connected by Programmable Routing Fabric

SPLD simple programmable logic device. ↳ add some FFs after a PAL or PLA

Hazards: inside Glitch: output



### HAZARDS

static & dynamic  
static-0 hazard: 0 → 0, but instead 0 → 1 → 0  
static-1 hazard: opposite of ↑  
Dynamic hazard: or

multi-level circuits

SR → tolerates momentary 00  
SR' → tolerates momentary 11

Generate function: always SET or RESET  
output glitches: if both are 0 or both 1, the don't-care has no choice

## DEFINITIONS

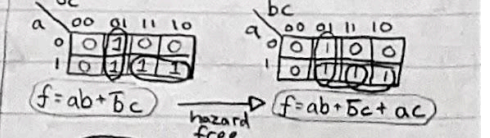
### PROCEDURE

- Analysis w/ Latches
- Label latch output, & feedback path
  - Logic equations for  $S_j$  and  $R_j$
  - [X] Must have to work correctly:  
 $S \cdot R = 0$  ← NOR (normal) NAND →  $S + R = 0$
  - Use known behaviour for outputs:  
 $Y = S + R \cdot y$  ← NOR NAND →  $Y = S + R \cdot y$
  - Make transition table
  - (perhaps) Make flow table

### Design

- Make Primitive Flow Table
- Reduce flow table
- Do State Assignment (no race conditions)
- Next & Output logic equations
- Draw circuit

If adjacent minterms aren't covered, a HAZARD EXISTS



for 2-level circuits, removing all static-1 hazards guarantees no hazards

(must have 0, or 1)

# ECE 124 Notes 7

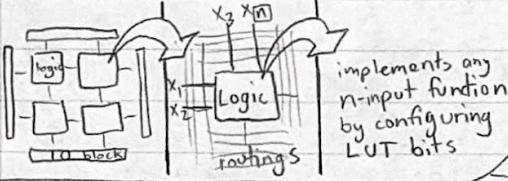
\*practice\*

- convert to JK FF (from state diagram)
- draw circuit out of state table
- Decoder, and memory stuff
- multiplexer system (w/enable)
- Counter system (w/load)
- Full Adder system
- merger diagrams
- partitioning (SA)
- critical races (RF)
- recognize a pattern
- ASM chart

## 28 FPGAs

Field Programmable Gate Arrays

Implemented with an LUT

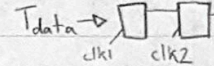


implements any n-input function by configuring LUT bits

asymmetric clock cycle == non-uniform duty cycle

inverted clocks make it harder to function

## 29 Timing Analysis



### Cycle Times

FF outputs take time to arrive back as inputs

To reach in time, our maximum frequency for 2 FFs is:

$$T_{\text{cycle}} \geq T_{\text{clock out}} + T_{\text{data}} + T_{\text{su}} + (T_{\text{clk1}} - T_{\text{clk2}})$$

logic between (FF2)

without violating the set-up time

clock skew (+ or -)

### Pins of a Chip

- su - cf → time when FF CLK INPUT active
- cc → CLK PIN
- df → data FF time
- dc → data PIN time
- tsu → su w.r.t. FF CLK (for FF input)
- th → FF hold time
- tsetup → PIN su
- thold → PIN hold



its a garbage meme but its kinda on point. you got this :)